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# Massive symmetric tensor field on AdS

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Abstract: The two-point Green function of a local operator in CFT corresponding to a massive symmetric tensor field on the AdS background is computed in the framework of the AdS/CFT correspondence. The obtained two-point function is shown to coincide with the two-point function of the graviton in the limit when the AdS mass vanishes.

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## Contents



# 1. Introduction

The AdS/CFT correspondence conjectured in [1] states that in the large $N$ limit and at large 't Hooft coupling $\lambda = g_{YM}^2 N$ the classical supergravity/M-theory on the Anti-de Sitter space (AdS) times a compact manifold is dual to a certain SU($N$) conformal gauge theory (CFT) defined on the boundary of AdS. One notable example of this correspondence is the duality between $D = 4$, $\mathcal{N} = 4$ supersymmetric Yang-Mills theory and $D = 10$ Type IIB supergravity theory on $AdS_5 \times S^5$. The precise formulation of the conjecture was given in [2, 3] where it was proposed to identify the generating functional for connected Green functions of local operators in CFT with the on-shell value of the supergravity action under the restriction that the supergravity fields satisfy the Dirichlet conditions on the boundary of AdS. Recall that in the standard representation of AdS as an upper-half space $x_0 > 0$, $x^k \in \mathbb{R}$, $k = 1, \ldots, d$ with the metric

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \frac{1}{x_0^2}\left(dx^0 dx^0 + dx^i dx^i\right),$$

the boundary includes the plane $x_0 = 0$ as well as the point $x_0 = \infty$. Since the boundary is located infinitely far away from any point in the interior the supergravity action is infrared divergent and must be regularized. As was pointed out in [4] the consistent regularization procedure (with respect to Ward identities) requires one to shift the boundary of AdS to the surface in the interior defined by $x_0 = \varepsilon$. Then the Dirichlet boundary value problem for supergravity fields is properly defined and one can compute the on-shell value of the supergravity action as a functional of the boundary fields. With the account of this regularization procedure the standard formulation of the AdS/CFT correspondence assumes the form:

$$\langle \mathcal{O}(\mathbf{x}_1) \cdots \mathcal{O}(\mathbf{x}_n) \rangle = \lim_{\varepsilon \to 0} \frac{\delta}{\delta \phi_1(\mathbf{x}_1)} \cdots \frac{\delta}{\delta \phi_n(\mathbf{x}_n)} S_{on-shell}\left(\phi_1(\mathbf{x}_1), \ldots, \phi_n(\mathbf{x}_n)\right)\Big|_{\substack{\phi_i(\mathbf{x}_i)=\phi_i \\ (x_0=\varepsilon, \mathbf{x}_i)}},$$

where $\mathbf{x}_i$ are some points on the boundary of $AdS_{d+1}$, $\mathcal{O}_i(\mathbf{x}_i)$ are gauge invariant composite operators in CFT and $\phi_i(\mathbf{x}_i)$ are the corresponding supergravity fields.

Here we used the convention in which the coordinate $x^\mu$ of $AdS_{d+1}$ is split according to: $x = (x_0, \mathbf{x})$, so that $\mathbf{x} \in \mathbb{R}^d$. The action $S_{on-shell}$ is the sum of the bulk supergravity action and the boundary terms necessary to make the AdS/CFT correspondence complete. The origin of these boundary terms was elucidated in [5] where it was shown that they appear in passing from the hamiltonian description of the bulk action to the lagrangean one.

The AdS/CFT correspondence has been tested by computing various two- and three-point functions of local operators in $D = 4$, $\mathcal{N} = 4$ supersymmetric Yang-Mills theory on $AdS_5$. In particular two-point functions corresponding to scalars [3]–[8], vectors [3, 4, 9, 10], spinors [10]–[12], the Rarita-Schwinger field [13]–[16], antisymmetric form fields [17]–[20] and the graviton [5, 21, 22] were computed on the AdS background. The only field in the supergravity spectrum [23] that has evaded the attention is the massive symmetric second-rank tensor field. In this paper we fill in the gap by computing the remaining two-point Green function.

We note that the Dirichlet boundary value problem for the massive symmetric tensor field is nontrivial due to the fact that the equations of motion for various components are coupled. Furthermore, in computing the two-point Green function we have to stick with the regularization procedure described above in order to obtain the consistent result. At the end of our computation we find that in the limit when the AdS mass vanishes the correlation function reduces to that of the massless symmetric tensor field, i.e. the graviton.

## 2. Equations of motion

The starting point in the calculation is the action for the symmetric second-rank tensor $\phi_{\mu\nu}$ on $AdS_{d+1}$ [24][1]:

$$\begin{aligned} S\left[\phi_{\mu\nu}\right] = \frac{1}{2\kappa^2}\int_{AdS} d^{d+1}x\sqrt{|g|}\Big(&-\frac{1}{4}\nabla_\lambda\phi_{\mu\nu}\nabla^\lambda\phi^{\mu\nu} + \frac{1}{4}\nabla_\lambda\phi\nabla^\lambda\phi - \\ &- \frac{1}{2}\nabla^\mu\phi^\nu_\mu\nabla_\nu\phi + \frac{1}{2}\nabla_\mu\phi^{\mu\nu}\nabla^\lambda\phi_{\lambda\nu} - \\ &- \frac{d}{2}\phi_{\mu\nu}\phi^{\mu\nu} + \frac{d}{4}\phi^2 - \frac{1}{4}m^2\phi_{\mu\nu}\phi^{\mu\nu} - \frac{1}{4}m^2\phi^2\Big)\,, \end{aligned} \tag{2.1}$$

where $g$ is the determinant of the AdS metric $g_{\mu\nu}$. Action (2.1) already contains the appropriate boundary terms and so can be readily used for computing the two-point Green function. The equations of motion following from (2.1) are [23, 24]:

$$\nabla^\lambda\nabla_\lambda\phi^\mu_\nu + (2 - m^2)\phi^\mu_\nu = 0\,, \tag{2.2}$$

$$\phi^\mu_\mu = 0\,, \qquad \nabla^\nu\phi^\mu_\nu = 0\,. \tag{2.3}$$

[1]Note that the AdS background satisfies restriction (10) of [24] and that to simplify our calculations we set the parameter $\xi$ of [24] equal to 1.

The massive terms in (2.1) destroys the standard symmetry $\delta\phi_{\mu\nu} = \nabla_\mu\xi_\nu + \nabla_\nu\xi_\mu$ which is present in the case of the massless symmetric tensor field. As a result of this symmetry breaking one can no longer perform gauge fixing. Since $\phi^\mu_\nu$ is traceless, we can eliminate the component $\phi^0_0$ from the equations of motion by using the constraint $\phi^0_0 + \phi^i_i = 0$. Let us introduce a concise notation:

$$\phi = \phi^i_i\,, \qquad \varphi^i = \phi^i_0\,, \qquad \varphi = \partial_i\varphi^i\,.$$

Then starting from (2.2) and (2.3) one can obtain the following system of coupled differential equations

$$\varphi - \left(\partial_0 - \frac{d+1}{x_0}\right)\phi = 0\,, \tag{2.4}$$

$$\left(\partial_0 - \frac{d+1}{x_0}\right)\varphi^i + \partial^k\phi^i_k = 0\,, \tag{2.5}$$

$$\left(\partial_0^2 - \frac{d+3}{x_0}\partial_0 + \frac{2(d+2)}{x_0^2} + \Box - \frac{m^2}{x_0^2}\right)\phi = 0\,. \tag{2.6}$$

Here eq. (2.6) corresponds to $\mu = \nu = 0$ in (2.2) while eqs. (2.4) and (2.5) follow from (2.3). To simplify the notation, we chose the convention in which indices are raised and lowered with the flat metric $\delta^{\mu\nu} = x_0^{-2}g^{\mu\nu}$ so that, in particular, $\Box = \partial_i\partial_j\delta^{ij}$. The Fourier mode solution for the field $\phi$ vanishing at $x_0 \to \infty$ is:

$$\phi(x_0, \mathbf{k}) = A(\varepsilon, \mathbf{k})\, x_0^{\frac{d}{2}+2}\, K_\Delta(x_0 k)\,, \tag{2.7}$$

where $k = |\mathbf{k}|$,

$$\Delta = \sqrt{\left(\frac{d}{2}\right)^2 + m^2}$$

and $K_\Delta$ is a modified Bessel function [25]:

$$K_\Delta(z) = \frac{\pi}{2}\frac{I_{-\Delta}(z) - I_\Delta(z)}{\sin\Delta\pi}\,, \qquad I_\Delta(z) = \sum_{k=0}^{\infty}\frac{1}{k!\Gamma(\Delta+k+1)}\left(\frac{z}{2}\right)^{\Delta+2k}.$$

Recall that modified Bessel functions satisfy the recurrence relations [25]:

$$K_{\Delta+1}(z) - K_{\Delta-1}(z) = \frac{2\Delta}{z}K_\Delta(z)\,, \qquad K_{\Delta+1}(z) + K_{\Delta-1}(z) = -2\frac{d}{dz}K_\Delta(z)\,. \tag{2.8}$$

By differentiating (2.5) with respect to $x_i$ and using (2.4) and (2.6) one gets:

$$\partial_i\partial^j\phi^i_j = \left(\frac{d-1}{x_0}\partial_0 + \Box - \frac{(d-1)(d+2)+m^2}{x_0^2}\right)\phi\,. \tag{2.9}$$

From (2.7) and (2.9) we can easily deduce the Fourier mode solution for the field

$$\pi(x_0, \mathbf{k}) \equiv k_i k^j \phi^i_j(x_0, \mathbf{k})\,,$$

$$\pi(x_0, \mathbf{k}) = A(\varepsilon, \mathbf{k})x_0^{\frac{d}{2}}\Big((d-1)(x_0k)K_{\Delta+1}(x_0k) + \Big(\Delta_-(\Delta_- - 1) + (x_0k)^2\Big)K_\Delta(x_0k)\Big),$$

where recurrence formulae (2.8) were used. Here we introduce a concise notation:

$$\Delta_- = \frac{d}{2} - \Delta\,, \qquad \Delta_+ = \frac{d}{2} + \Delta\,.$$

Taking the ratio of $\phi$ and $\pi$ at $x_0 = \varepsilon$ results in the following relation

$$\pi(\mathbf{k}) = \varepsilon^{-2}\,\frac{(d-1)(\varepsilon k)K_{\Delta+1}(\varepsilon k) + \left(\Delta_-(\Delta_- -1) + (\varepsilon k)^2\right) K_\Delta(\varepsilon k)}{K_\Delta(\varepsilon k)}\phi(\mathbf{k})\,,$$

where $\pi(\mathbf{k}) = \pi(\varepsilon, \mathbf{k})$ and, similarly, for other fields. In the limit $\varepsilon \to 0$, $\pi(\mathbf{k}) \sim \varepsilon^{-2}\phi(\mathbf{k})$ and therefore if we keep $\pi(\mathbf{k})$ fininte as $\varepsilon \to 0$, then $\phi(\mathbf{k})$ will tend to zero. On the other hand, keeping $\phi(\mathbf{k})$ finite leads to the divergence of $\pi(\mathbf{k})$. Consequently, we ought to fix $\pi$ at $x_0 = \varepsilon$. Thus we have:

$$\phi(x_0, \mathbf{k}) = \frac{x_0^{\frac{d}{2}+2}}{\varepsilon^{\frac{d}{2}}}\frac{K_\Delta(x_0 k)}{(d-1)(\varepsilon k)K_{\Delta+1}(\varepsilon k) + (\Delta_-(\Delta_- -1) + (\varepsilon k)^2)K_\Delta(\varepsilon k)}\pi(\mathbf{k}), \tag{2.10}$$

$$\pi(x_0, \mathbf{k}) = \frac{x_0^{\frac{d}{2}}}{\varepsilon^{\frac{d}{2}}}\frac{(d-1)(x_0 k)K_{\Delta+1}(x_0 k) + \left(\Delta_-(\Delta_- -1) + (x_0 k)^2\right) K_\Delta(x_0 k)}{(d-1)(\varepsilon k)K_{\Delta+1}(\varepsilon k) + \left(\Delta_-(\Delta_- -1) + (\varepsilon k)^2\right) K_\Delta(\varepsilon k)}\pi(\mathbf{k}). \tag{2.11}$$

The Fourier mode solution for the field $\varphi$ is found by substituting (2.10) into (2.4):

$$\varphi(x_0, \mathbf{k}) = \frac{x_0^{\frac{d}{2}+1}}{\varepsilon^{\frac{d}{2}}}\frac{(1-\Delta_-)K_\Delta(x_0 k) - (x_0 k)K_{\Delta+1}(x_0 k)}{(d-1)(\varepsilon k)K_{\Delta+1}(\varepsilon k) + \left(\Delta_-(\Delta_- -1) + (\varepsilon k)^2\right) K_\Delta(\varepsilon k))}\pi(\mathbf{k})\,, \tag{2.12}$$

where once again recurrence formulae (2.8) were used.

Next we turn our attention to the field $\varphi_i$. Here we need to use equation

$$\left(\Box - \frac{m^2}{x_0^2}\right)\varphi^i + \frac{2}{x_0}\partial^i\phi = \partial_0\partial^k\phi^i_k\,, \tag{2.13}$$

that follows from $(i0)$-component of (2.2) by taking into account (2.5). Now we decompose $\varphi_i$ into the transversal and longitudinal parts:

$$\varphi_i^\perp = \varphi_i - \frac{\partial_i}{\Box}\varphi\,.$$

Rewriting (2.5) and (2.13) for the transversal part of $\varphi_i$ leads to:

$$\left(\Box - \frac{m^2}{x_0^2}\right)\varphi_i^\perp = \partial_0\left(\partial_k\phi_i^k - \frac{\partial_i\partial_l\partial^m}{\Box}\phi_m^l\right)\,, \tag{2.14}$$

$$\left(\partial_0 - \frac{d+1}{x_0}\right)\varphi_i^\perp = -\left(\partial_k\phi_i^k - \frac{\partial_i\partial_l\partial^m}{\Box}\phi_m^l\right)\,. \tag{2.15}$$

By differentiating (2.15) with respect to $x_0$ and taking into account (2.14) we obtain an homogeneous differential equation for the field $\varphi_i^\perp$:

$$\left(\partial_0^2 - \frac{d+1}{x_0}\partial_0 + \frac{d+1}{x_0^2} + \Box - \frac{m^2}{x_0^2}\right)\varphi_i^\perp = 0\,. \tag{2.16}$$

The Fourier mode solution is given by:

$$\varphi_i^{\perp}(x_0,\mathbf{k}) = B_i(\varepsilon,\mathbf{k})\, x_0^{\frac{d}{2}+1}\, K_\Delta(x_0k)\,. \tag{2.17}$$

To find $B_i(\varepsilon,\mathbf{k})$ substitute (2.17) into (2.15) to obtain the following formula

$$k_l\phi^l_i(x_0,\mathbf{k}) - \frac{k_i}{k^2}\pi(x_0,\mathbf{k}) = iB_i(\varepsilon,\mathbf{k})x_0^{\frac{d}{2}}\Big(\Delta_- K_\Delta(x_0k) + (x_0k)K_{\Delta+1}(x_0k)\Big)\,. \tag{2.18}$$

Combining (2.17) and (2.18) we find:

$$k^l\phi^i_l(\mathbf{k}) - \frac{k^i}{k^2}\pi(\mathbf{k}) = i\varepsilon^{-1}\frac{\Delta_- K_\Delta(\varepsilon k) + (\varepsilon k)K_{\Delta+1}(\varepsilon k)}{K_\Delta(\varepsilon k)}\,\varphi_i^{\perp}(\mathbf{k})\,.$$

Using the same arguments as before we find that in order to avoid the divergence at $\varepsilon \to 0$ the solutions for the fields $\varphi_i^{\perp}$ and $k_l\phi^l_i - \frac{k_i}{k^2}\pi$ should take the following form

$$\varphi_i^{\perp}(x_0,\mathbf{k}) = -i\frac{x_0^{\frac{d}{2}+1}}{\varepsilon^{\frac{d}{2}}}\frac{K_\Delta(x_0k)}{\Delta_- K_\Delta(\varepsilon k) + (\varepsilon k)K_{\Delta+1}(\varepsilon k)}\left(k^l\phi^i_l(\mathbf{k}) - \frac{k^i}{k^2}\pi(\mathbf{k})\right)\,, \tag{2.19}$$

$$k^l\phi^i_l(x_0,\mathbf{k}) - \frac{k^i}{k^2}\pi(x_0,\mathbf{k}) = \frac{x_0^{\frac{d}{2}}}{\varepsilon^{\frac{d}{2}}}\frac{\Delta_- K_\Delta(x_0k) + (x_0k)K_{\Delta+1}(x_0k)}{\Delta_- K_\Delta(\varepsilon k) + (\varepsilon k)K_{\Delta+1}(\varepsilon k)}\left(k^l\phi^i_l(\mathbf{k}) - \frac{k^i}{k^2}\pi(\mathbf{k})\right). \tag{2.20}$$

Setting $\mu = i$, $\nu = j$ in (2.3) and taking into account (2.3) we arrive at the following equation

$$\left(\partial_0^2 - \frac{d-1}{x_0}\partial_0 + \Box - \frac{m^2}{x_0^2}\right)\phi^i_j = \frac{2}{x_0}\left(\partial_j\varphi^i + \partial^i\varphi_j\right) + \frac{2\delta^i_j}{x_0}\phi\,. \tag{2.21}$$

Let us introduce the transversal traceless part of $\phi^i_j$:

$$\begin{aligned}\bar{\phi}^i_j(x_0,\mathbf{k}) = {}& \phi^i_j - \frac{\partial^i\partial_k}{\Box}\phi^k_j - \frac{\partial_j\partial^k}{\Box}\phi^i_k + \frac{\partial^i\partial_j\partial_k\partial^m}{\Box^2}\phi^k_m + \\ &+\frac{1}{d-1}\left(\frac{\partial^i\partial_j}{\Box} - \delta^i_j\right)\left(\phi - \frac{\partial_k\partial^m}{\Box}\phi^k_m\right)\,.\end{aligned} \tag{2.22}$$

Rewriting (2.21) for the transversal traceless part yields:

$$\left(\partial_0^2 - \frac{d-1}{x_0}\partial_0 + \Box - \frac{m^2}{x_0^2}\right)\bar{\phi}^i_j = 0\,. \tag{2.23}$$

The Fourier mode solution of eq. (2.23) is

$$\bar{\phi}^i_j(x_0,\mathbf{k}) = \frac{x_0^{\frac{d}{2}}}{\varepsilon^{\frac{d}{2}}}\frac{K_\Delta(x_0k)}{K_\Delta(\varepsilon k)}\bar{\phi}^i_j(\mathbf{k})\,. \tag{2.24}$$

Taking into account (2.10), (2.11), (2.20), (2.22) and (2.24) we obtain:

$$
\begin{aligned}
\phi^i_j(x_0,\mathbf{k}) = \frac{x_0^{\Delta_-}}{\varepsilon^{\Delta_-}}\Bigg[&\frac{\mathcal{K}_\Delta(x_0k)}{\mathcal{K}_\Delta(\varepsilon k)}\bar{\phi}^i_j(\mathbf{k}) + \frac{\Delta_-\mathcal{K}_\Delta(x_0k)+\mathcal{K}_{\Delta+1}(x_0k)}{\Delta_-\mathcal{K}_\Delta(\varepsilon k)+\mathcal{K}_{\Delta+1}(\varepsilon k)}\times \\
&\times\left(\frac{k^ik_l}{k^2}\phi^l_j(\mathbf{k}) + \frac{k_jk^l}{k^2}\phi^i_l(\mathbf{k}) - 2\frac{k^ik_j}{k^4}\pi(\mathbf{k})\right)+ \\
&+\frac{(d-1)\mathcal{K}_\Delta(x_0k)+\left(\Delta_-(\Delta_- -1)+(x_0k)^2\right)\mathcal{K}_\Delta(x_0k)}{(d-1)\mathcal{K}_\Delta(\varepsilon k)+\left(\Delta_-(\Delta_- -1)+(\varepsilon k)^2\right)\mathcal{K}_\Delta(\varepsilon k)}\frac{k^ik_j}{k^4}\pi(\mathbf{k}) - \\
&-\Bigg(\frac{(x_0k)^2\mathcal{K}_\Delta(x_0k)}{(d-1)(\varepsilon k)\mathcal{K}_{\Delta+1}(\varepsilon k)+\left(\Delta_-(\Delta_- -1)+(\varepsilon k)^2\right)\mathcal{K}_\Delta(\varepsilon k)} - \\
&\quad-\frac{(d-1)\mathcal{K}_\Delta(x_0k)+\left(\Delta_-(\Delta_- -1)+(x_0k)^2\right)\mathcal{K}_\Delta(x_0k)}{(d-1)\mathcal{K}_\Delta(\varepsilon k)+\left(\Delta_-(\Delta_- -1)+(\varepsilon k)^2\right)\mathcal{K}_\Delta(\varepsilon k)}\Bigg)\times \\
&\times\frac{1}{d-1}\left(\frac{k^ik_j}{k^2}-\delta^i_j\right)\frac{\pi(\mathbf{k})}{k^2}\Bigg],
\end{aligned}
\tag{2.25}
$$

while taking into account (2.12) and (2.19) gives:

$$
\begin{aligned}
\varphi_i(x_0,\mathbf{k}) = -i\frac{x_0^{\Delta_-+1}}{\varepsilon^{\Delta_-}}\Bigg[&\frac{\mathcal{K}_\Delta(x_0k)}{\Delta_-\mathcal{K}_\Delta(\varepsilon k)+\mathcal{K}_{\Delta+1}(\varepsilon k)}\left(k_l\phi^l_i(\mathbf{k}) - \frac{k_i}{k^2}\pi(\mathbf{k})\right) - \\
&-\frac{(1-\Delta_-)\mathcal{K}_\Delta(x_0k)-\mathcal{K}_{\Delta+1}(x_0k)}{(d-1)\mathcal{K}_{\Delta+1}(\varepsilon k)+\left(\Delta_-(\Delta_- -1)+(\varepsilon k)^2\right)\mathcal{K}_\Delta(\varepsilon k)}\frac{k_i}{k^2}\pi(\mathbf{k})\Bigg],
\end{aligned}
\tag{2.26}
$$

where we introduced a concise notation: $\mathcal{K}_\Delta(z) = z^\Delta K_\Delta(z)$.

## 3. Two-point Green function

To compute the Green function in the framework of the AdS/CFT correspondence we need to evaluate the on-shell value of the action. Taking into account equations of motion (2.2) and (2.3) one finds that the on-shell value of (2.1) is

$$
S_{on-shell} = \frac{\varepsilon^{-d+1}}{8\kappa^2}\int\limits_{x_0=\varepsilon} d^dx\left(\phi^i_j\partial_0\phi^j_i - \phi\partial_0\phi + 2\phi\varphi - 2\varphi^k\partial_i\phi^i_k + \frac{2(d+1)}{\varepsilon}\left(\phi^2+\varphi^k\varphi_k\right)\right). \tag{3.1}
$$

Let us first consider the contribution to $S_{on-shell}$ that depends locally on boundary fields, i.e. does not contain the normal derivative $\partial_0$. We expect that such terms do not contribute to the non-local part of $S_{on-shell}$. So we need to consider the behavior of $\phi$ and $\varphi^k$ on the boundary of AdS. To this end we note that according to (2.10) the field $\phi(\mathbf{k})$ is local since

$$
\frac{K_\Delta(x_0k)}{(d-1)(\varepsilon k)K_{\Delta+1}(\varepsilon k)+(\Delta_-(\Delta_- -1)+(\varepsilon k)^2)K_\Delta(\varepsilon k)} = \frac{1}{\Delta_+(\Delta_+-1)} + O(\varepsilon^2k^2)\,, \tag{3.2}
$$

while expanding solution (2.27) in $\varepsilon$ gives:

$$\varphi^i(\mathbf{k}) = -(i\varepsilon)\left[\left(\frac{1}{\Delta_+}+O(\varepsilon^2k^2)\right)\left(k^l\phi^i_l(\mathbf{k})-\frac{k^i}{k^2}\pi(\mathbf{k})\right)+\right.$$
$$\left.+\left(\frac{1}{\Delta_+}+O(\varepsilon^2k^2)\right)\frac{k^i}{k^2}\pi(\mathbf{k})\right]. \qquad (3.3)$$

Here we restricted our attention to terms of order $O(\varepsilon^2k^2)$ and lower. In deriving (3.3) use was made of the power series expansion of the modified Bessel function [25]:

$$\mathcal{K}_\Delta(z) = 2^{\Delta-1}\Gamma(\Delta)\left(1+\frac{z^2}{4(1-\Delta)}+\cdots\right), \qquad (3.4)$$

where ellipsis indicates terms of order $z^4$ and higher which evidently lead to local expressions as well as terms of order $z^{2\Delta+2}$ which become negligible in the limit $\varepsilon\to 0$. From expression (3.3) it follows that $\varphi_i(\mathbf{k})$ is a local field. Consequently, those terms in (3.1) that depend only on the value of fields on the boundary do not contribute to the non-local part of $S_{on-shell}$ as we expected.

Next we consider terms with the normal derivative $\partial_0$. In evaluating such terms it is useful to employ the following identity

$$\frac{d}{dx_0}\left(\left(\frac{x_0}{\varepsilon}\right)^\gamma\frac{F(x_0k)}{F(\varepsilon k)}\right)\Bigg|_{x_0=\varepsilon} = \frac{\gamma}{\varepsilon}+\frac{k}{\varepsilon}\frac{d}{dk}\ln F(k\varepsilon). \qquad (3.5)$$

Taking into account solution (2.10), identity (3.5) and expansion (3.4) we find that $\partial_0\phi(\varepsilon,\mathbf{x})$ is equal to:

$$\partial_0\phi(\varepsilon,\mathbf{x}) = \int\frac{d^dk}{(2\pi)^d}e^{-i\mathbf{kx}}\left(\frac{\Delta_-+2}{\varepsilon}+O(\varepsilon k^2)\right)\varepsilon^2\left(\frac{1}{\Delta_+(\Delta_+-1)}+O(\varepsilon^2k^2)\right)\pi(\mathbf{k}). \qquad (3.6)$$

Clearly $\partial_0\phi$ is a local expression and therefore does not contribute to the non-local part of $S_{on-shell}$. Consequently, the non-local part is entirely determined by the following expression

$$\frac{\varepsilon^{-d+1}}{8\kappa^2}\int d^dx\,\phi^j_i(\mathbf{x})\int\limits_{x_0=\varepsilon}\frac{d^dk}{(2\pi)^d}e^{-i\mathbf{kx}}\partial_0\phi^i_j(x_0,\mathbf{k}). \qquad (3.7)$$

Taking into account solution (2.25), identity (3.5) and expansion (3.4) we see that the expression for $\partial_0\phi^i_j$ gets three different contributions: the first contribution comes from differentiating the ratio $x_0/\varepsilon$ raised to the power $\Delta_-$ or $\Delta_-+2$; the second contribution comes from $O(\varepsilon k^2)$ terms in the power expansion of the logarithmic derivative of various functions in (2.25) and the third contribution comes from $O(\varepsilon^{2\Delta-1}k^{2\Delta})$

terms in the same expansion. From (2.25) we can easily read off the first contribution which is equal to

$$\frac{\Delta_-}{\varepsilon}\bar{\phi}^i_j(\mathbf{k}) + \frac{\Delta_-}{\varepsilon}\left(\frac{k^i k_l}{k^2}\phi^l_j(\mathbf{k}) + \frac{k_j k^l}{k^2}\phi^i_l(\mathbf{k}) - 2\frac{k^i k_j}{k^4}\pi(\mathbf{k})\right) + \frac{\Delta_-}{\varepsilon}\frac{k^i k_j}{k^4}\pi(\mathbf{k}) - \\ -\frac{1}{d-1}\left(\frac{k^i k_j}{k^2} - \delta^i_j\right)\left(\frac{\Delta_- + 2}{\varepsilon}\frac{(\varepsilon k)^2}{\Delta_+(\Delta_+ - 1)} - \frac{\Delta_-}{\varepsilon}\right)\frac{\pi(\mathbf{k})}{k^2} = \\ = -\frac{2}{d-1}\frac{k^i k_j}{k^2}\frac{\varepsilon}{\Delta_+(\Delta_+ - 1)}\pi(\mathbf{k}) + \text{ local}, \tag{3.8}$$

where we took into account the expansion of $\phi^i_j$ into the transversal and longitudinal parts given by (2.22). Next $O(\varepsilon k^2)$ terms from the expansion of the logarithmic derivative give:

$$\frac{\varepsilon k^2}{2}\left[\frac{1}{1-\Delta}\bar{\phi}^i_j(\mathbf{k}) + \frac{\Delta_+ - 2}{\Delta_+(1-\Delta)}\left(\frac{k^i k_l}{k^2}\phi^l_j(\mathbf{k}) + \frac{k_j k^l}{k^2}\phi^i_l(\mathbf{k}) - 2\frac{k^i k_j}{k^4}\pi(\mathbf{k})\right) + \right. \\ + \frac{(\Delta_+ - 2)(\Delta_+ - 3)}{(1-\Delta)\Delta_+(\Delta_+ - 1)}\frac{k^i k_j}{k^4}\pi(\mathbf{k}) - \frac{1}{d-1}\left(\frac{k^i k_j}{k^2} - \delta^i_j\right) \times \\ \left. \times\left(\frac{(\varepsilon k)^2}{(1-\Delta)\Delta_+(\Delta_+ - 1)} - \frac{(\Delta_+ - 3)(\Delta_+ - 2)}{(1-\Delta)\Delta_+(\Delta_+ - 1)}\right)\frac{\pi(\mathbf{k})}{k^2}\right] = \\ = \frac{2}{d-1}\frac{k^i k_j}{k^2}\frac{\varepsilon}{\Delta_+(\Delta_+ - 1)}\pi(\mathbf{k}) + \text{ local}, \tag{3.9}$$

where we used (2.22) and (3.4). Adding together (3.8) and (3.9) the non-local part cancels and we are led to a purely local expression which does not contribute to the non-local part of $S_{on-shell}$. Finally, $O(\varepsilon^{2\Delta-1}k^{2\Delta})$ terms in the expansion of the logarithmic derivative give:

$$\eta\varepsilon^{2\Delta-1}k^{2\Delta}\left[\bar{\phi}^i_j(\mathbf{k}) + \frac{\Delta_-}{\Delta_+}\left(\frac{k^i k_l}{k^2}\phi^l_j(\mathbf{k}) + \frac{k_j k^l}{k^2}\phi^i_l(\mathbf{k}) - 2\frac{k^i k_j}{k^4}\pi(\mathbf{k})\right) + \right. \\ \left. + \frac{\Delta_-(\Delta_- - 1)}{\Delta_+(\Delta_+ - 1)}\frac{k^i k_j}{k^4}\pi(\mathbf{k}) + \frac{1}{d-1}\left(\frac{k^i k_j}{k^2} - \delta^i_j\right)\frac{\Delta_-(\Delta_- - 1)}{\Delta_+(\Delta_+ - 1)}\frac{\pi(\mathbf{k})}{k^2}\right] = \\ = \eta\,\varepsilon^{2\Delta-1}k^{2\Delta}\left[\phi^i_j(\mathbf{k}) - \frac{2\Delta}{\Delta_+}\left(\frac{k^i k_l}{k^2}\phi^l_j(\mathbf{k}) + \frac{k_j k^l}{k^2}\phi^i_l(\mathbf{k})\right) - \frac{4\Delta(1-\Delta)}{\Delta_+(\Delta_+ - 1)}\frac{k^i k_j}{k^4}\pi(\mathbf{k})\right], \tag{3.10}$$

where

$$\eta = -\frac{\Delta}{2^{2\Delta-1}}\frac{\Gamma(1-\Delta)}{\Gamma(1+\Delta)}.$$

In the process of deriving (3.10) we dropped all terms containing $\delta^i_j$ since such terms are negligible in the limit $\varepsilon \to 0$. To understand why this is so, note that when $\delta^i_j$ is contracted with $\phi^j_i(\mathbf{x})$ from (3.7) it gives the trace of $\phi^i_j(\mathbf{x})$ which according to (2.10) and (3.2) is order $O(\varepsilon^2)$ at $x_0 = \varepsilon$.

Now putting together (3.7) and (3.10) we arrive at the following formula

$$
\begin{aligned}
S_{on-shell}[\phi^i_j] = \frac{\eta\varepsilon^{2\Delta-d}}{8\kappa^2} \int\int d^d x\, d^d y\, \mathbf{\Phi}^j_i(\mathbf{x})\, \mathbf{\Phi}^r_s(\mathbf{y}) \int \frac{d^d k}{(2\pi)^d} e^{-i\mathbf{k}(\mathbf{x}-\mathbf{y})}\, k^{2\Delta} \times \\
\times \left[ \frac{1}{2}\delta^i_r \delta^s_j + \frac{1}{2}\delta^{is}\delta_{jr} - \frac{\Delta}{\Delta_+}\left( \frac{k^i k_r}{k^2}\delta^s_j + \frac{k^i k^s}{k^2}\delta_{jr} + \frac{k_j k^s}{k^2}\delta^i_r + \frac{k_j k_r}{k^2}\delta^{is} \right) + \right. \\
+ \frac{4\Delta(\Delta-1)}{\Delta_+(\Delta+-1)} \frac{k^i k_j k_r k^s}{k^4} + \frac{2\Delta}{\Delta_+(\Delta_+ - 1)} \times \\
\left. \times \left( + \frac{\Delta_- - \Delta_+^2}{2d\Delta}\delta^i_j \delta^s_r + \frac{k_r k^s}{k^2}\delta^i_j + \frac{k^i k_j}{k^2}\delta^s_r \right) \right], \qquad (3.11)
\end{aligned}
$$

where we introduced the traceless part of $\phi^i_j$:

$$
\mathbf{\Phi}^i_j(\mathbf{x}) = \phi^i_j(\mathbf{x}) - \frac{\delta^i_j}{d}\phi(\mathbf{x}) .
$$

In order to complete the calculation of the two-point Green function we need to evaluate the integral over $\mathbf{k}$ in (3.11). To this end, we employ the standard formula for the Fourier transform of generalized functions [26]:

$$
\int \frac{d^d k}{(2\pi)^d} e^{-i\mathbf{k}\mathbf{x}} \frac{k^{i_1} k^{i_2} \cdots k^{i_n}}{k^n} |\mathbf{k}|^\delta = \frac{2^{\delta-n}\Gamma\left(\frac{d+\delta-n}{2}\right)}{\pi^{d/2}\Gamma\left(\frac{-\delta+n}{2}\right)} (i)^n \partial_{i_1}\partial_{i_2}\cdots\partial_{i_n} \frac{1}{|\mathbf{x}|^{d+\delta-n}} . \qquad (3.12)
$$

With the help of (3.12) we find that the non-local part of the on-shell value of the action is equal to:

$$
\begin{aligned}
S_{on-shell}[\phi^i_j] = \frac{C_{d,\Delta}\varepsilon^{2\Delta-d}}{2} \int\int d^d x\, d^d y \frac{\mathbf{\Phi}^j_i(\mathbf{x})\mathbf{\Phi}^r_s(\mathbf{y})}{|\mathbf{x}-\mathbf{y}|^{2\Delta+d}} \times \\
\times \left[ \frac{1}{2} J^i_r(\mathbf{x}-\mathbf{y}) J^s_j(\mathbf{x}-\mathbf{y}) + \frac{1}{2} J^{is}(\mathbf{x}-\mathbf{y}) J_{jr}(\mathbf{x}-\mathbf{y}) - \frac{1}{d}\delta^i_j \delta^r_s \right],
\end{aligned}
$$

where we introduced

$$
J^i_j(\mathbf{x}) = \delta^i_j - 2\frac{x^i x_j}{|\mathbf{x}|^2} \qquad \text{and} \qquad C_{d,\Delta} = \frac{\Delta(\Delta_+ + 1)\Gamma(\Delta_+)}{2\pi^{\frac{d}{2}}\kappa^2(\Delta_+ - 1)\Gamma(\Delta)} .
$$

From this we can easily deduce the two-point function of local operators in the boundary CFT corresponding to the massive symmetric traceless second rank tensor field $\mathbf{\Phi}^i_j$:

$$
\begin{aligned}
\langle \mathcal{O}^i_j(\mathbf{x}) \mathcal{O}^s_r(\mathbf{y}) \rangle = \frac{C_{d,\Delta}}{|\mathbf{x}-\mathbf{y}|^{2\Delta+d}} \left[ \frac{1}{2} J^i_r(\mathbf{x}-\mathbf{y}) J^s_j(\mathbf{x}-\mathbf{y}) + \right. \\
\left. + \frac{1}{2} J^{is}(\mathbf{x}-\mathbf{y}) J_{jr}(\mathbf{x}-\mathbf{y}) - \frac{1}{d}\delta^i_j \delta^s_r \right] . \qquad (3.13)
\end{aligned}
$$

Here we performed rescaling in order to remove the regularization parameter $\varepsilon$. Note that in the limit $m^2 \to 0$ or, equivalently, $\Delta \to d/2$ and $\Delta_+ \to d$ the obtained expression correctly reproduces the two-point function corresponding to the massless symmetric tensor field (graviton) [5, 21, 22].

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